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STABILITY ANALYSIS IN ELASTIC STATES OF VERY SLENDER RODS FIXED BY ONE END WITH STRESSES AND STRAINS ANALYSIS AS EXEMPLIFIED BY CYLINDRICAL SHAPED PLYWOOD MADE OF BIRCH

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Abstract. The theory of axially compressed cylindrical shaped plywood stability is presented in the paper. The differential equation of the elastic line, its slope, the critical stress and the shell stresses and strains of very slender plywood tube in elastic states are considered in the paper under conditions of flat cross-sections and little slope of the elastic line. The received theoretical results of critical stresses are related to slenderness ratio and to product of the radius and the wall thickness of the cylindrical shaped plywood made of birch. The received theoretical results of stresses and strains are presented as an example of the cylindrical shaped plywood.

Key words: stability, critical force, stress, strain, cylindrical shaped, plywood

INTRODUCTION

The basic theory of slender rod losing stability in elastic states was formulated by Euler [1744, 1759]. The author introduced the concept of critical load and gave, according to his theory, the formulas for the differential equation of the elastic line and for critical stress of axially compressed rod:

$$EJ\frac{d^2y}{dx^2} = P_{cr}y \qquad \sigma_{cr}^{Euler} = \left(\frac{\pi}{\lambda}\right)^2 E \tag{1}$$

where: P_{cr} – the critical force,
E – Young's modulus of elasticity of rod,
J – moment of inertia of cross-section area,

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σ_{cr} – critical stress,
λ – slenderness ratio.

This theory was developed during next years by Grashof [1878], Considère [1889], Salmon [1921], Bleich [1952], Timoshenko and Gere [1963], Březina [1966], Vol'mir [1967], Życzkowski [1981], Grigoljuk and Kabanov [1987], Weiss and Giżejowski [1991], Reese and Wriggers [1995] and many others. It was also developed especially for wood [Yoshihara et al. 1998, Kudela and Slaninka 2002 and others].

STABILITY ANALYSIS

The theory presented in this paper was already described by the author [Murawski 2003 a, b] for rod compressed by balls. Similarly to that papers [Murawski 1992, 1998, 2002 a] by the author it has been assumed in the analysis of rods stability that the state of stresses in the critical transverse section, after losing stability and before losing carrying capacity, is the result of a superposition of pure compression and bending (Fig. 1)

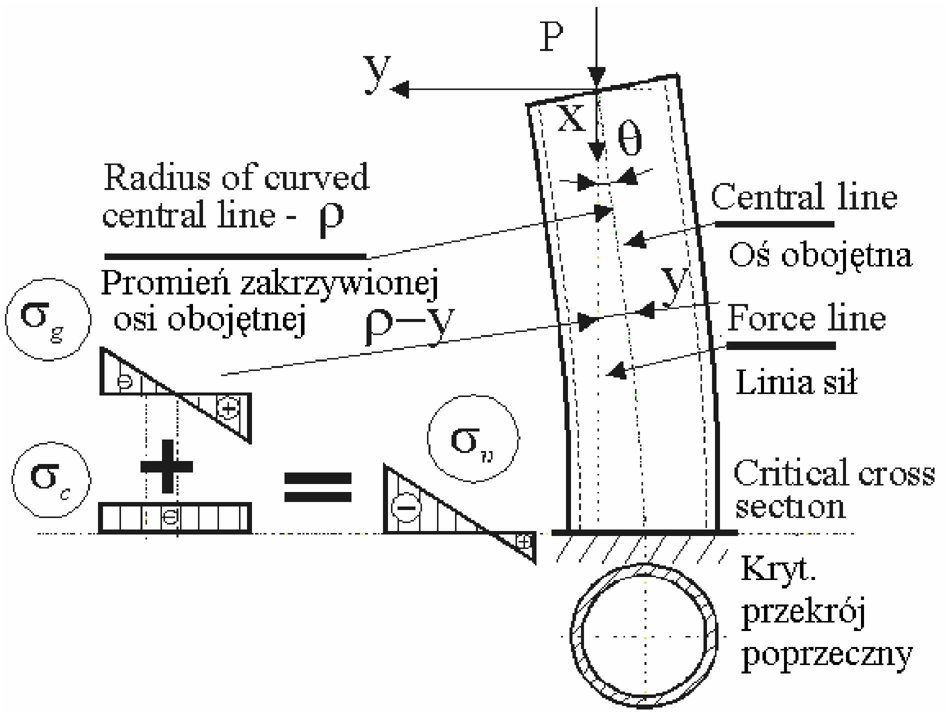

Fig. 1. Stresses in the critical transverse section of an axially compressed tube, after loss of stability according to the author's hypothesis

Rys. 1. Naprężenia w krytycznym przekroju poprzecznym osiowo ściskanej rury, po utracie stateczności według hipotezy autora and the start of losing of a carrying capacity (the maximum force on a force-shortness graph) in elastic states follows after the exit of the resultant neutral layer from the critical transverse section. In this paper it has been assumed as a simplification for very slender columns, that the start of loss of a carrying capacity follows after the exit forces line from the critical transverse section.

Like in [Murawski 2003 a] the extension of any fibre at the distance y from central layer and the stress in this fibre is:

$$\varepsilon = \frac{\pm(\rho + y)\theta - \rho\theta}{\rho\theta} \qquad \sigma_n = \sigma_g - \sigma_c = \pm\frac{y}{\rho}E_{\parallel} - \frac{P}{A} \tag{2}$$

where: ρ – the radius of the curved central line,
θ – angle of the central line slope in relation to the forces line (Fig. 1),
$E_{\parallel(\perp)}$ – Young's modulus of elasticity of a rod in parallel (orthogonal) direction to wood fibres,
P – axial force,
A – area of a constant cross-section.

The differential equations of the elastic line and of its slope and equation of the elastic line, for boundary conditions: when $x = L$ then $dy/dx = 0$ and when $x = 0$ then $y = 0$, are:

$$\frac{4E_{II}R^2t}{ln(2\pi Rt)}\frac{d^2y}{dx^2} = P \qquad \frac{4E_{II}R^2t}{ln(2\pi Rt)}\frac{dy}{dx} = P(x - L) \qquad \frac{4E_{II}R^2t}{ln(2\pi Rt)}y = Px\left(\frac{x}{2} - L\right) \tag{3}$$

where: R – the median tube radius,
t – wall thickness,
L – tube length.

From the assumption, that the losing of carrying capacity follows when the force line exit outside the critical transverse section ($y_{x=L} = R$), the critical stress is equal to:

$$\sigma_{cr}^E = \frac{8E_{II}}{\lambda^2\pi\, ln(2\pi\varphi)} \tag{4}$$

where: $\phi = R \cdot t$.

The conclusion of the function (4) showed in Fig. 2 is that to determine σ_{cr}^E for all cases of cylinders is not enough to relate them only to the slenderness ratio λ, but we must relate them to $\phi = R \cdot t$ too, because for the same slenderness ratio λ (like in the Euler's formula) we may get different values σ_{cr}^E (depending on ϕ).

The analysis of the theoretical results shows in Fig. 2, that the simplifications of this theory (taking into consideration only normal stresses, the assumption of flat cross-sections and a little slope) limit use of it in engineering practice only to very slender rods with big $\phi = R \cdot t$ value. One of the limits for cylindrical shaped plywood is $\phi = Rt > 0.16$ because in that case the function (4) is asymptotic going to plane $\phi = 0.16$. Because the whole phenomenon occurs in the elastic states, then for birch plywood a slenderness ratio $\lambda > \lambda_{ltd} = 127.6$. If we take the slenderness ratio with referred length $L_{ref} = 2L$ for the critical stress formula like in [Murawski 2003 a], we would get the surface function exactly like (4).

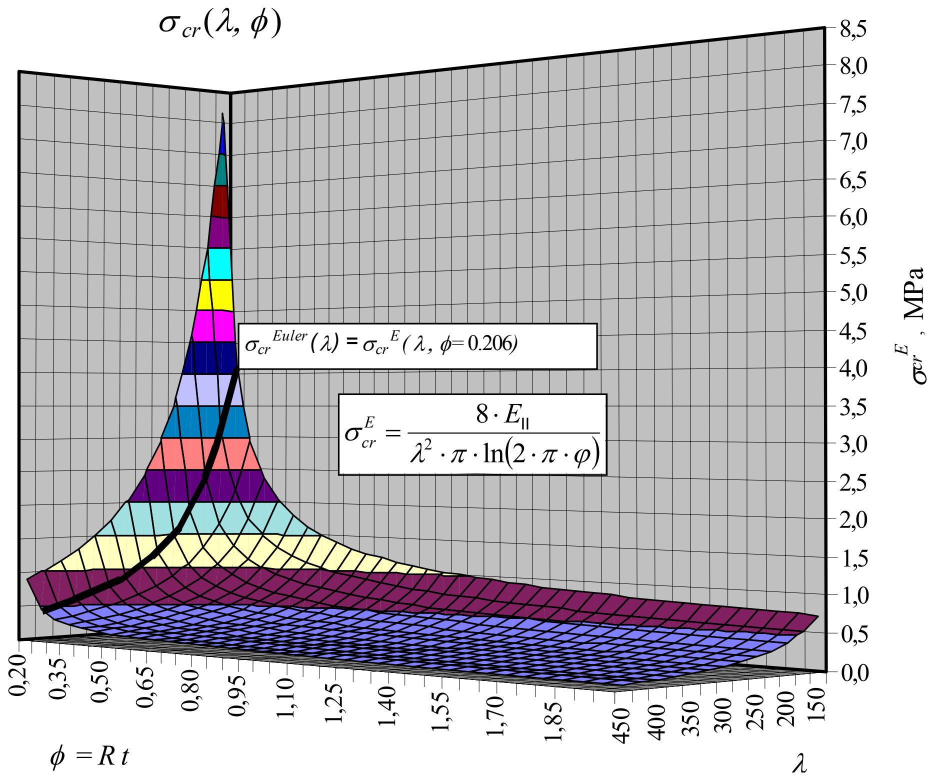

Fig. 2. The surface function σ_{cr}^{E} (λ,ϕ) of the cylindrical shaped plywoods made of birch fixed by one end

Rys. 2. Funkcja powierzchniowa σ_{cr}^{E} (λ,ϕ) dla cylindrycznych kształtek sklejkowych wykonanych z brzozy, utwierdzonych jednostronnie

Despire the limits, below is presented (Fig. 3-5) the stability analysis (the function of the elastic line *y(x),* its slope *dy/dx,* the dependence $y_{L/2}(P)$) of the cylindrical birch plywood with R = 0.75 m, t = 0.5 m, L = 80 m, ϕ = 0.375, λ = 143.1 and ϕ = 0.375 as the theoretical example. In that case $\sigma_{cr}^{Euler}(\lambda)$ = 1.988 MPa but σ_{cr}^{E} (λ,ϕ) = 2.154 MPa. The plywood tube would be made of birch according to the author's idea [Murawski 2002 b] (Fig. 6; for which assumed [Krzysik 1978], that $R_{c\,\parallel}$ = 43 MPa and the Young's modulus $E_{\parallel}$ = 16500 MPa – the cylindrical shaped plywood would be rolled up with fibres parallel to the axis).

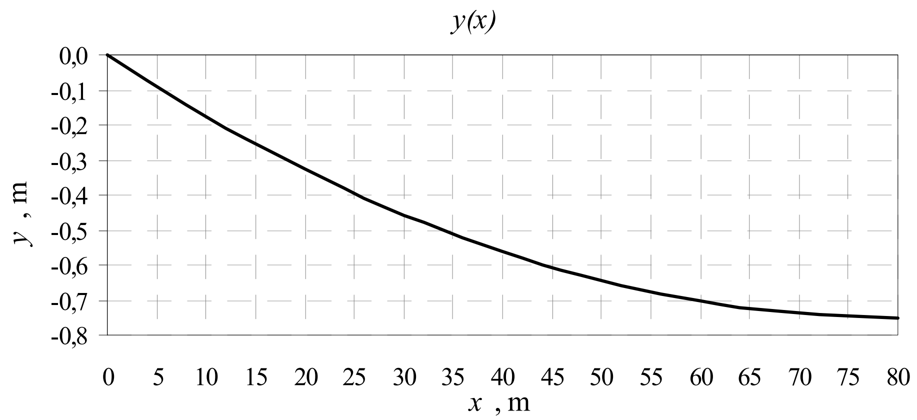

Fig. 3. The function *y(x)* of the elastic line of cylindrical plywood made of birch with R = 0.75 m, t = 0.5 m, L = 80 m
Rys. 3. Funkcja *y(x)* ugięcia osi obojętnej cylindrycznej kształtki sklejkowej wykonanej z brzozy o R = 0,75 m, t = 0,5 m, L = 80 m

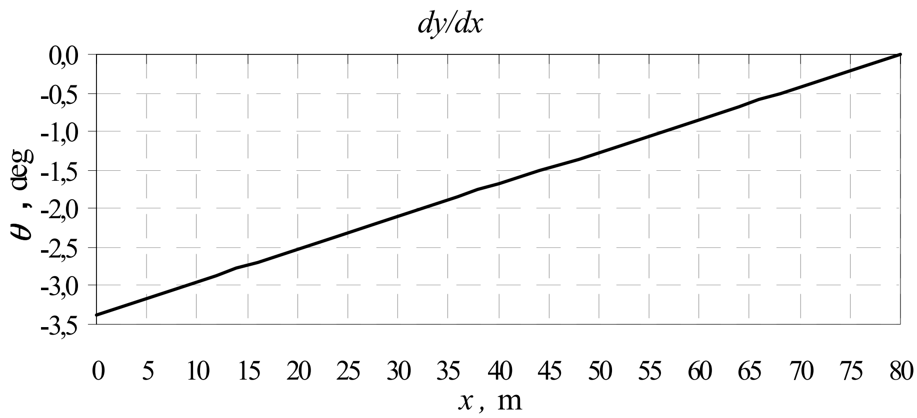

Fig. 4. The function *dy/dx* of the elastic line slope of plywood made of birch with R = 0.75 m, t = 0.5 m, L = 80 m
Rys. 4. Funkcja *dy/dx* nachylenia osi obojętnej cylindrycznej kształtki sklejkowej wykonanej z brzozy o R = 0,75 m, t = 0,5 m, L = 80 m

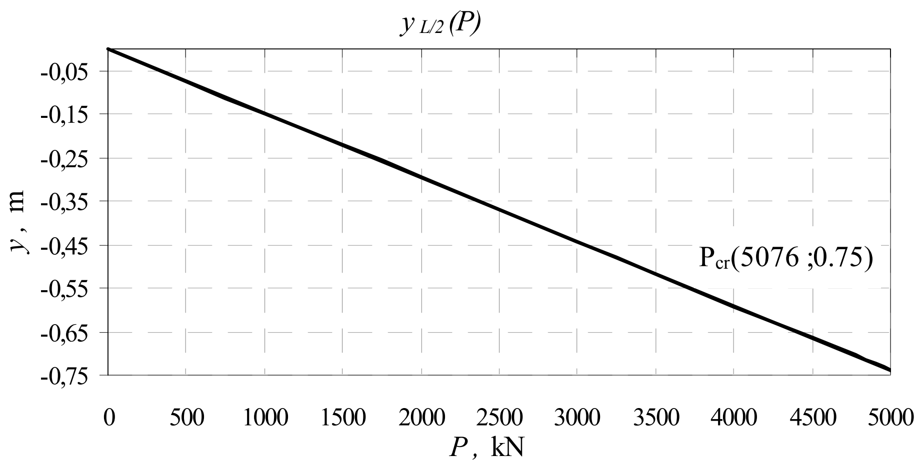

Fig. 5. The function $y_{L/2}(P)$ of cylindrical plywood made of birch with R = 0.75 m, t = 0.5 m, L = 80 m
Rys. 5. Funkcja $y_{L/2}(P)$ dla cylindrycznej kształtki sklejkowej wykonanej z brzozy o R = 0,75 m, t = 0,5 m, L = 80 m

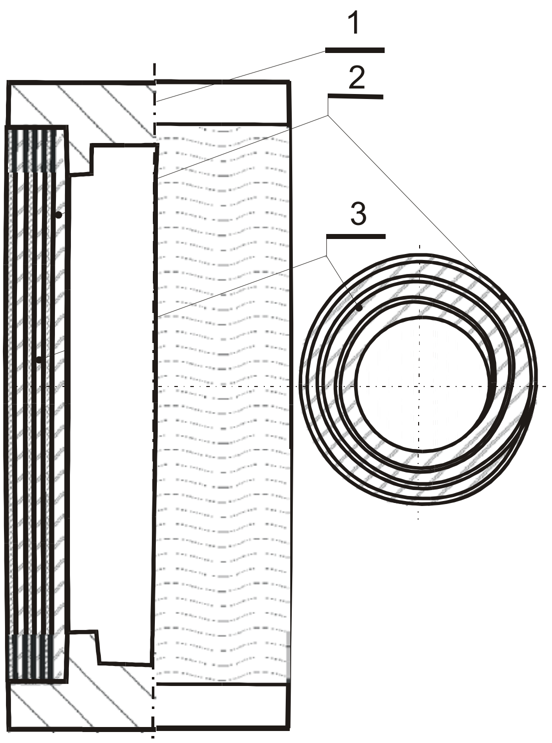

Fig. 6. The example of the plywood tube made according to the author's idea [Murawski 2002] (patent application includes not only slender tubes); 1 – the closing element, 2 – the shell, 3 – the insert
Rys. 6. Przykład rury sklejkowej wykonanej według pomysłu autora [Murawski 2002] (zgłoszenie patentowe obejmuje nie tylko smukłe rury); 1 – element zamykający, 2 – powłoka, 3 – wkładka

STRESSES AND STRAINS ANALYSIS

It has been assumed that the state of strains is the result of a superposition of bending and pure compression strains. The extension of any fibre at the distance *y* from the central layer and the stress in this fibre is (Fig. 7):

$$\varepsilon_n = \pm\varepsilon_g - \varepsilon_c = \pm\frac{y}{\rho} - \frac{\Delta L}{L} \tag{5}$$

where: ε_g – the bending strain,
ε_c – compression strain,
ΔL – tube length increment.

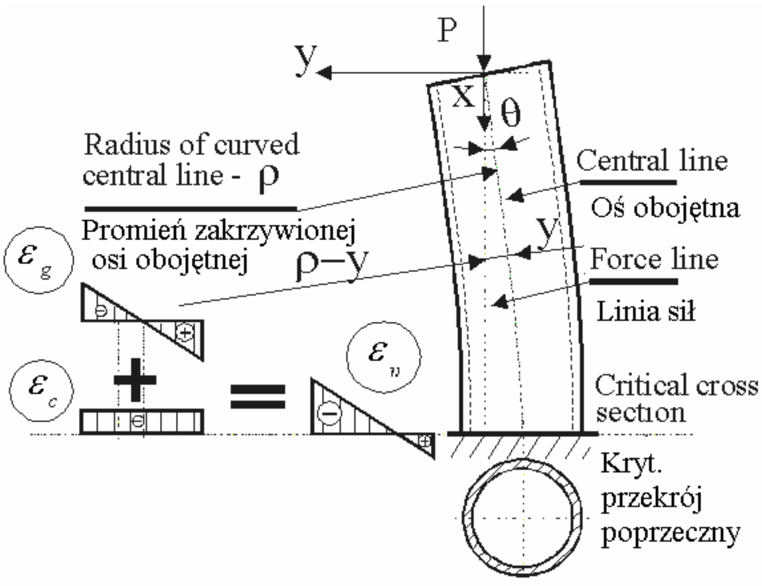

Fig. 7. Strains in the critical transverse section of axially compressed cylindrical shaped plywood, after losing stability and before losing carrying capacity, according to author's own hypothesis
Rys. 7. Odkształcenia względne w krytycznym przekroju poprzecznym osiowo ściskanej rury, po utracie stateczności według hipotezy autora

From the equilibrium of moments, the shell stresses σ_n depending on *x* and *y* are:

$$\sigma_n(x,y) = \frac{yx\left(\frac{x}{2} - L\right) ln(2\pi Rt) P^2}{4\pi E_{II} R^5 t^2} - \frac{P}{2\pi Rt} \tag{6}$$

The formulas to determine σ_g – the bending stress, σ_c – the compression stress, ε_g and ε_c caused by external load as well as σ_y – the orthogonal stress and ε_y – the orthogonal strain caused with ν (Poisson's ratio) by internal load, are like follows:

$$\sigma_c(x,y) = \frac{\sigma_g}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} \approx \sigma_g(x,y) \qquad \varepsilon_c(x,y) = \frac{\varepsilon_g}{\sqrt{1+\left(\frac{dy}{dx}\right)^2}} \approx \varepsilon_g(x,y) \tag{7}$$

$$\sigma_y(x,y) \approx -\sigma_n \nu \frac{E_{\perp}}{E_{II}} \qquad \varepsilon_n(x,y) = \frac{\sigma_n(x,y)}{E_{II}} \qquad \varepsilon_y(x,y) = -\varepsilon_n \nu \tag{8}$$

Below are presented (Fig. 8-11) the stresses and strains analysis in the shell as a theoretical example (assumed: $E_{\perp}$ = 1200 MPa, ν = 0.425).

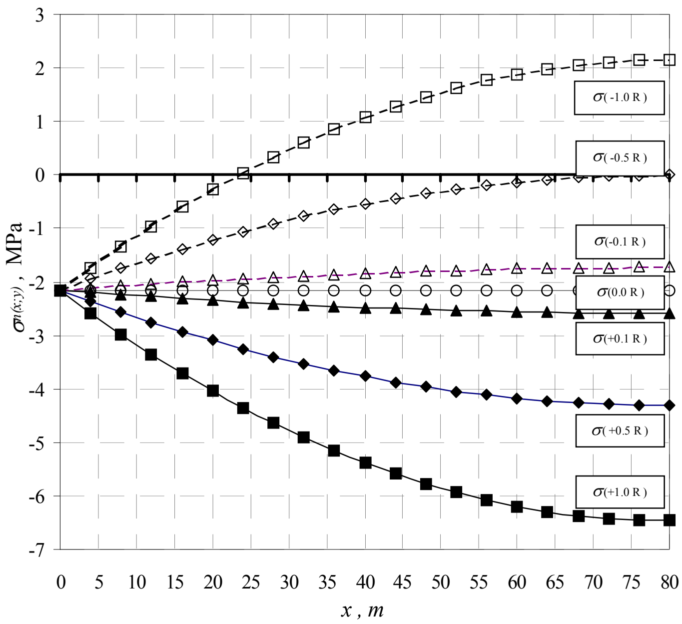

Fig. 8. Values of stresses $\sigma_n(x,y)$ in the shell of cylindrical plywood made of birch with R = 0.75, t = 0.5, L = 80 m

Rys. 8. Wartości naprężeń $\sigma_n(x,y)$ w powłoce cylindrycznej kształtki sklejkowej wykonanej z brzozy o R = 0,75, t = 0,5, L = 80 m

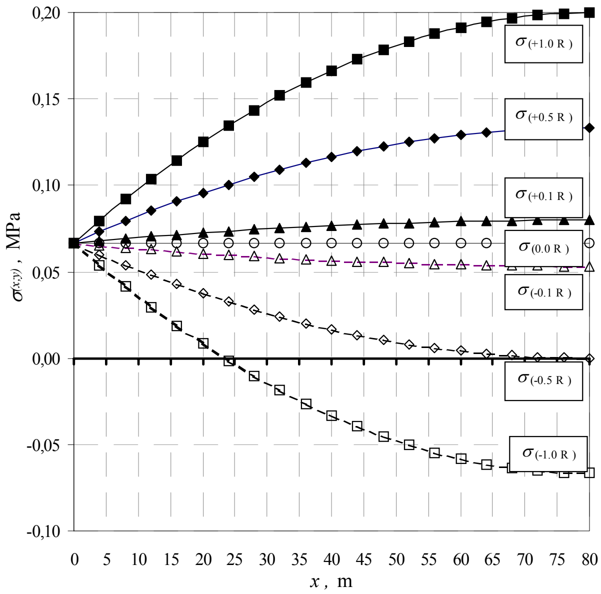

Fig. 9. Values of stresses $\sigma_y(x,y)$ in the shell of cylindrical plywood made of birch with R = 0.75, t = 0.5, L = 80 m

Rys. 9. Wartości naprężeń $\sigma_y(x,y)$ w powłoce cylindrycznej kształtki sklejkowej wykonanej z brzozy o R = 0,75, t = 0,5, L = 80 m

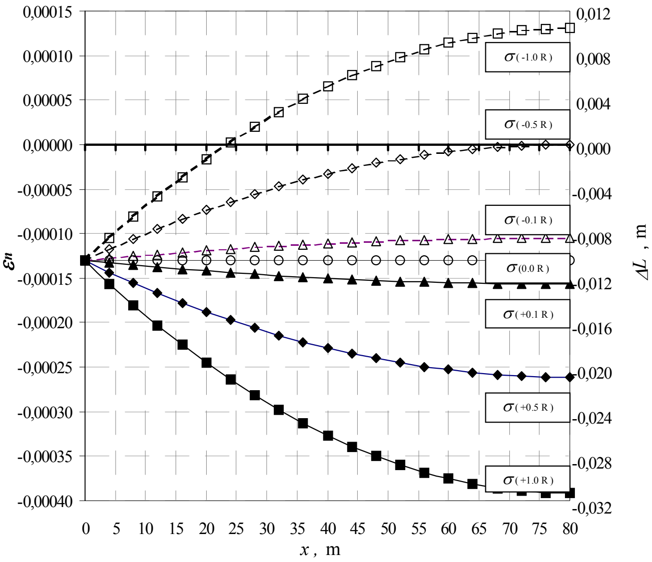

Fig. 10. Values of strains $\varepsilon_n(x,y)$ in the shell of cylindrical plywood made of birch with R = 0.75, t = 0.5, L = 80 m

Rys. 10. Wartości odkształceń $\varepsilon_n(x,y)$ w powłoce cylindrycznej kształtki sklejkowej wykonanej z brzozy o R = 0,75, t = 0,5, L = 80 m

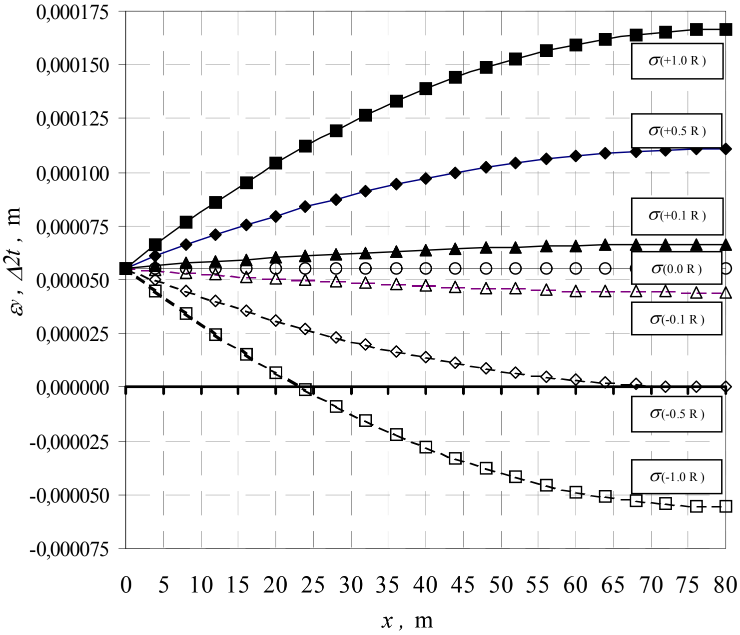

Fig. 11. Values of strains $\varepsilon_y(x,y)$ in the shell of cylindrical plywood made of birch with $R = 0.75$, $t = 0.5$, $L = 80$ m

Rys. 11. Wartości odkształceń $\varepsilon_n(x,y)$ w powłoce cylindrycznej kształtki sklejkowej wykonanej z brzozy o $R = 0{,}75$, $t = 0{,}5$, $L = 80$ m

REFERENCES

Bleich F., 1952. Buckling strength of metal structures. McGraw-Hill Book Company Inc. New York.

Březina V., 1966. Stateczność prętów konstrukcji metalowych. Arkady Warszawa.

Considère A., 1889. Resistance des pièces comprimèes. Congrès international de procedès de costruction.

Euler L., 1744. Methodus inveniendi lineas curves maximi minimive proprietate gaudentes. Appendix 1. De curvis elasticis (in Latin). Lausanne and Geneva.

Euler L., 1759. Sur la force de colonnes (in French). Memoires de l'Academie de Berlin, 13, 251--282.

Grashof F., 1878. Theorie der Elasticität und Festigkeit. Berlin.

Grigoljuk E., Kabanov B.B., 1987. Ustojichivost obolochek. Leningrad.

Kudela J., Slaninka R., 2002. Stability of wood columns loaded in Buckling. Part 1. Centric Buckling. Drev. Vysk. 47(2), 19-34.

Krzysik F., 1978. Nauka o drewnie. PWN Warszawa.

Murawski K., 1992. Stability of thin shell columns in elasto-plastic states. (Stateczność prętów cienkościennych w stanach sprężysto-plastycznych). In: Mechanika, 14 Międzynarodowe Sympozjum Naukowe Studentów i Młodych Pracowników Nauki, Zielona Góra 6-9.04.1992, ZP WSI, 38-43.

Murawski K., 1998. The Modelling of Energy Consuming Process in Layered Vehicles Bumper (Modelowanie procesu pochłaniania energii w warstwowych zderzakach). Maszyn. Rozpr. Dokt. Poznań.
Murawski K., 2002 a. The Engesser-Shanley modified theory of thin-walled cylindrical rods with example of use for steel St35. Acta Sci. Pol. Architectura. 1-2 (1-2), 85-95.
Murawski K., 2002 b. Patent application: Shaped plywood P 353556 (22.04.2002).
Murawski K., 2003 a. Stability analysis in elastic states of very slender cylindrical shaped plywood compressed by balls. Ann. Warsaw Agric. Univ. SGGW-AR, For Wood Technol. 53, 257-260.
Murawski K., 2003 b. Stresses and strains analysis in elastic states of very slender cylindrical shaped plywood compressed by balls. Ann. Warsaw Agric. Univ. SGGW-AR, For Wood Technol. 53, 247-251.
Reese S., Wriggers P., 1995. A Finite-Element Method for Stability Problems in Finite Elasticity. Int. J. Num. Meth. Eng. 38, 7, 1171-1200.
Salmon E.H., 1921. Columns. Oxford Technical Publication London.
Timoshenko S.P., Gere J.M., 1963. Teoria stateczności sprężystej. Arkady Warszawa.
Weiss S., Giżejowski M., 1991. Stateczność konstrukcji metalowych. Układy prętowe. Arkady Warszawa.
Vol'mir A.C., 1967. Ustojichivost deformiruemytch sistem. Nauka Moskwa.
Yoshihara H., Ohta M., Kubojima Y., 1998. Prediction of the buckling stress of intermediate wooden columns using the secant modulus. J Wood Sci. 44, 69-72.
Życzkowski M., 1981. Podstawy analizy stateczności prętów sprężystych. Współczesne metody analizy stateczności konstrukcji. Ossolineum Wrocław.

ANALIZA STATECZNOŚCI W STANACH SPRĘŻYSTYCH BARDZO SMUKŁYCH PRĘTÓW UTWIERDZONYCH W JEDNYM KOŃCU Z ANALIZĄ NAPRĘŻEŃ I ODKSZTAŁCEŃ NA PRZYKŁADZIE CYLINDRYCZNEJ KSZTAŁTKI SKLEJKOWEJ WYKONANEJ Z BRZOZY

Streszczenie. W pracy przedstawiono teorię stateczności osiowo ściskanej cylindrycznej kształtki sklejkowej. Rozpatrywano równanie różniczkowe osi obojętnej i jej nachylenia, krytyczne naprężenia, naprężenia i odkształcenia w powłoce dla bardzo smukłych sklejkowych rur w stanach sprężystych przy założeniu hipotezy płaskich przekrojów i małych ugięć osi obojętnej. Otrzymane teoretyczne wyniki naprężeń krytycznych odniesiono do smukłości cylindrycznej kształtki sklejkowej wykonanej z brzozy oraz do iloczynu promienia przez grubość ścianki. Otrzymane teoretyczne wyniki wartości naprężeń i odkształceń zaprezentowano używając przykładowej cylindrycznej kształtki sklejkowej.

Słowa kluczowe: stateczność, siła krytyczna, naprężenie, odkształcenie, cylindryczna kształtka, sklejka

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