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ASSESSMENT OF FUNCTIONS DESCRIBING THE DEPENDENCE OF DIAMETER AT BREAST HEIGHT AND TREE HEIGHT IN STANDS ON THE TRANSFORMATION OF FOREST MANAGEMENT TO SELECTION FOREST IN THE SUDETEN

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ABSTRACT

In this study 13 functions describing the dependence between diameter at breast height and tree height were tested to determine stand volumes as affected by the changes in the forest management in the Sudeten Mts. Tree measurements were recorded in two sample plots in the years 2013 and 2015. The functions were tested on data concerning spruce, beech, sycamore maple and fir. Function parameters were determined through transformation of equations to the linear or polynomial form and solution of the system of normal equations, as well as the Levenberg-Marquardt iterative method applied in the Statistica 12 software package. Fitting of function curves to the data was better when applying the iterative method. The best fit was found for the graph of the 2[nd] degree parabola. A good fit was obtained when using the functions proposed by Korsuň (1935) ($H = \exp(a + b \cdot \ln(D) + c \cdot \ln^2(D))$) and by Prodan (1951) ($H - 1.3 = D^2/(a + b \cdot D + c \cdot D^2)$). Goodness of fit for the other tested functions was much lower. Despite a very good fit, graphs of the 2[nd] degree parabola and the Korsuň function do not meet all the postulates of dendrometry imposed on curves of tree height. For this reason the function proposed by Prodan was selected for practical use.

Keywords: height-diameter curves, approximation, numerical iterative method

INTRODUCTION

The determination of the regression relationship between diameter at breast height and tree height is crucial in view of its applications in forestry practice. For instance, volume tables for single trees, volume increment tables and yield tables are constructed based on this dependence (Bruchwald, 1970; Meixner, 1964; Trampler, 1974). This dependence is also used in studies on forest dynamics (Pretzsch, 2009). In contrast to diameter at breast height, tree height measurement is relatively difficult and time-consuming, for this reason height is measured only for some trees in the stand. Height of the other trees is read from regression equations. Regression equations should thus well reflect the power and character of this relationship (Rymer-Dudzińska, 1978a). However, in practice this dependency varies in each stand and depends on the species, age, habitat, geographical location, stand age structure, tree dimensions, damage, etc. (Rymer-Dudzińska, 1973; Lebocký and Petráš, 2015; Loetsch et al., 1973; Petráš et al., 2014; Schmidt et al., 2011). For this reason it is strictly local in character. In view of the above, in literature on the subject starting from

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a study by Näslund (1929) new functions have been proposed, which graphs may be height curves, while previously proposed functions have been repeatedly tested (e.g. Adamec, 2015; El Mamoun et al., 2013; Gómez-García et al., 2015; Huang et al., 1992; Kaźmierczak and Grala-Michalak, 2006; Rymer-Dudzińska, 1978a; Stankova and Diéguez-Aranda, 2013).

In order to construct yield and volume increment tables many attempts have been made to develop uniform height tables, in which depending on stand characteristics constant parameters are assumed for height curves, of which one is the age of stand (Bruchwald, 1993; Bruchwald and Rymer-Dudzińska, 1981; Bruchwald and Witkowska, 1993; Bruchwald and Wirowski, 1993; Bruchwald and Wróblewski, 1994; Bruchwald et al., 1994; Rymer-Dudzińska, 1994; Rymer-Dudzińska, 1978b).

The greatest number of publications cited above concerns even-aged, one-storeyed stands, with the predominance of one species. Another problem is connected with the construction of height curves for mixed stands (Petráš et al., 2014) and for uneven-aged stands, particularly those managed as selection forests. A model proposed by Prodan was developed specifically for such stands (Prodan 1944, cited after: Prodan, 1951). Studies conducted by the author on the transformation of the management system to selection forest in the Sudeten Mts. showed the need to find a potentially optimal model of height curves for the established sample plots.

METHODS

Tree measurements were taken in experimental sites of the Department of Silviculture, the Poznań University of Life Sciences. One of them was the control unit in Trzebieszowice, the Lądek Zdrój Forest District, compartment 46, in a mountain forest site (LG). Measurements were taken in 2015. Analysed data comprised measurements of Norway spruce, European beech and sycamore maple trees. The other control unit, Chełmsko, is located in the Kamienna Góra Forest District, compartment 334, at 600–670 m a.s.l., in a mountain mixed forest site (LMG). Measurements were taken in 2013. Analyses were conducted on measurement data for spruce, beech and silver fir trees. Dark data were analysed in each case.

A short taxation description of the examined area is placed in Table 1.

Table 1. A short taxation description of examined control units

Trait	Trzebieszowice	Chełmsko
1	2	3
Altitude, m	470–570	620–670
Soil type	brown	podsolic brown
Mean temperature of, °C:		
January	–1.6	–4.0
July	16.5	10.2
year	6.7	6.4
Annual precipitation, mm	550	825
Type of forest site	mixed mountain forest and mountain forest (LMG and LG)	mixed mountain forest (LMG)

Table 1 – cont. / Tabela 1 – cd.

1	2	3
Stand volume, $m^3 \cdot ha^{-1}$	450	381
Share of species, %:		
spruce	37	60
beech	26	16
Mean height, m:		
spruce	41	27
beech	21	21
Mean DBH, cm:		
spruce	40	24
beech	20	16

Following preliminary selection a total of 13 functions, which were selected for analyses, are specified in Table 1. They were collected from literature on dendrometry, although functions 4 and 13 have broader applications and functions 6 and 11 were constructed within the field of econometry.

Empirical parameters of the functions were established using two methods. The first, referred to as analytical, consisted in a transformation of the equation leading to a simple equation of a straight line (a function with two parameters) or an equation of a 2nd degree parabola (a function with three parameters), followed by the solution of the system of standard equations. The other method, purely numerical, consisted in the application of the iterative method (Levenberg-Marquardt), used by the Statistica 12 software.

Goodness of fit was measured using the coefficient of determination and the coefficient of random variation.

The coefficient of determination was calculated using the formula:

$$R^2 = \frac{\sum(\hat{y}_i - \bar{y})^2}{\sum(y_i - \bar{y})^2}$$

This coefficient informs what part of total variation is explained by regression. The higher its value, the better goodness of fit of the equation to empirical data is found.

In order to calculate the coefficient of random variation it is necessary first to calculate the residual variance:

$$S_e^2 = \frac{\sum(y_i - \hat{y}_i)^2}{n-k}$$

and then to use it in the calculation of the coefficient of random variation:

$$W_e = \frac{S_e}{\bar{y}} \cdot 100\%$$

The lower the value of that coefficient is, the better the goodness of fit of the equation to empirical data. In all the above formulas the symbols denote:

y_i – the measured value of the *i*-th explained variable
$\hat{y}_i$ – the expected value of the *i*-th explained variable
$\bar{y}$ – the mean value of the explained variable calculated based on measured values
n – the number of observations
k – the number of estimated parameters
$S_e = \sqrt{S_e^2}$ – standard deviation of residuals.

Additionally, in order to verify whether values of the functions do not tend to accumulate above or below the data, the Wald-Wolfowitz runs test was conducted for all the functions.

RESULTS

Obtained coefficients of determination and coefficients of random variation for the six sets of data and the two methods used to determine function parameters are given in Tables 2–7. A comparison of the two methods of determining empirical parameters indicates that the iterative method is more accurate. It provides higher coefficients of determination and lower coefficients of random variation. Functions 9, 10 and 11 are exceptions in this respect, as identical results were obtained using both methods. In the case of the binomial (function 13) although the coefficients of determination are identical for both methods, the coefficients of random variation are lower for the iterative method. No marked differences were observed in the goodness of fit of the function graphs to the data. The Wald-Wolfowitz runs test showed no trend in any of the analysed cases for function curves to be positioned above or below the data. Functions adopted from econometry produce similar values as the others. However, in order to identify those providing the best goodness of fit a ranking of functions was created,

Table 2. Functions of one variable which graphs were used as height curves

No.	Name of function	Equation	Source
1	Michailoff	$H = ae^{-k/D} + 1.3$	Michajloff, 1943
2	Petterson 1	$H = \left(\frac{D}{a+bD}\right)^3 + 1.3$	Petterson, 1955
3	Petterson 2	$H = \left(\frac{1}{a+bD}\right)^{2.5} + 1.3$	Petterson, 1955
4	Oliveira	$H = e^{a+b/D}$	Gadow and Bredenkamp, 1992
5	Näslund	$H = \left(\frac{D}{a+bD}\right)^2 + 1.3$	Näslund, 1929
6	Törnquist (type 1)	$H = a\left(\frac{D}{b+D}\right)$	Stanisz, 1993
7	Prodan	$H = \frac{D^2}{a+bD+cD^2} + 1.3$	Prodan, 1944 (after Prodan, 1951)
8	Korsuň 1	$H = e^{a+b\ln D+c\ln 2D}$	Korsuň, 1935
9	Korsuň 2	$H = a + b\ln D$	Korsuň, 1935
10	Hyperbolic	$H = a + b\frac{1}{D}$	Rymer-Dudzińska, 1973
11	Pawłowski	$H = a\sqrt{\log(D+1)} + b$	Pawłowski, 1978
12	Levaković	$H = a\left(\frac{D}{1+D}\right)^b + 1.3$	Levaković, 1935
13	binomial	$H = a + bD + cD^2$	–

Table 3. Coefficients of determination R^2 and coefficients of random variation W_e measuring goodness of fit for function graphs to empirical data for Norway spruce in the Trzebieszowice control unit

No.	Provided by the analytical method		Provided by the iterative method	
	R^2	W_e	R^2	W_e
1	0.7600	11.22	0.7788	10.93
2	0.7646	11.19	0.7695	11.03
3	0.6294	14.15	0.7673	11.09
4	0.7739	11.04	0.7800	10.91
5	0.7541	11.40	0.7637	11.30
6	0.7131	12.19	0.7477	11.68
7	0.7859	10.79	0.7909	10.63
8	0.7925	10.69	0.7937	10.56
9	0.7588	11.42	0.7588	11.42
10	0.7337	12.00	0.7337	12.00
11	0.7665	11.24	0.7665	11.24
12	0.7624	11.32	0.7778	10.96
13	0.7869	10.86	0.7869	10.73

Table 4. Coefficients of determination R^2 and coefficients of random variation W_e measuring goodness of fit for function graphs to empirical data for European beech in the Trzebieszowice control unit

No.	Provided by the analytical method		Provided by the iterative method	
	R^2	W_e	R^2	W_e
1	0.7887	18.12	0.7999	17.73
2	0.8085	17.27	0.8099	17.29
3	0.8031	17.41	0.8111	17.23
4	0.7858	18.25	0.7972	17.86
5	0.8102	17.16	0.8125	17.17
6	0.8070	17.17	0.8132	17.14
7	0.8074	17.26	0.8139	17.11
8	0.8128	17.23	0.8141	17.10
9	0.8125	17.17	0.8125	17.17
10	0.7332	20.48	0.7332	20.48
11	0.8100	17.29	0.8100	17.29
12	0.7924	17.96	0.8021	17.64
13	0.8115	17.35	0.8115	17.22

Table 5. Coefficients of determination R^2 and coefficients of random variation W_e measuring goodness of fit for function graphs to empirical data for sycamore maple in the Trzebieszowice control unit

No.	Provided by the analytical method		Provided by the iterative method	
	R^2	W_e	R^2	W_e
1	0.6950	19.00	0.7397	17.67
2	0.7670	16.67	0.7712	16.57
3	0.7064	18.46	0.7769	16.36
4	0.6915	19.14	0.7313	17.96
5	0.7834	16.06	0.7850	16.06
6	0.8065	15.12	0.8081	15.17
7	0.8223	14.64	0.8398	13.88
8	0.8071	15.51	0.8276	14.39
9	0.7900	15.89	0.7900	15.89
10	0.6287	21.13	0.6287	21.13
11	0.7767	16.38	0.7767	16.38
12	0.7063	18.65	0.7483	17.38
13	0.8368	14.29	0.8368	14.00

Table 6. Coefficients of determination R^2 and coefficients of random variation W_e measuring goodness of fit for function graphs to empirical data for Norway spruce in the Chełmsko control unit

No.	Provided by the analytical method		Provided by the iterative method	
	R^2	W_e	R^2	W_e
1	0.8719	10.01	0.8738	10.14
2	0.8656	10.33	0.8680	10.27
3	0.8625	10.46	0.8662	10.34
4	0.8711	10.12	0.8737	10.05
5	0.8607	10.49	0.8633	10.45
6	0.8421	11.11	0.8487	10.99
7	0.8855	9.65	0.8884	9.44
8	0.8814	9.82	0.8896	9.39
9	0.8554	10.75	0.8554	10.75
10	0.8166	12.11	0.8166	12.11
11	0.8596	10.59	0.8596	10.59
12	0.8721	10.09	0.8052	12.48
13	0.8880	9.56	0.8880	9.46

Table 7. Coefficients of determination R^2 and coefficients of random variation W_e measuring goodness of fit for function graphs to empirical data for European beech in the Chełmsko control unit

No.	Provided by the analytical method		Provided by the iterative method	
	R^2	W_e	R^2	W_e
1	0.8130	23.23	0.8817	18.61
2	0.9005	17.30	0.9018	17.01
3	0.8533	20.32	0.9037	16.85
4	0.8027	23.90	0.8715	19.41
5	0.9041	16.78	0.9056	16.70
6	0.8978	17.20	0.9011	17.09
7	0.9036	17.02	0.9056	16.70
8	0.8908	18.19	0.9056	16.70
9	0.8796	18.87	0.8796	18.87
10	0.6946	30.05	0.6946	30.05
11	0.8687	19.71	0.8687	19.71
12	0.8246	22.50	0.8854	18.33
13	0.9139	16.19	0.9139	15.95

as presented in Table 8. The ranking list comprises six sets of data, two methods to estimate parameters, two measures of goodness of fit and thirteen functions. The minimum total of ranks for a function is thus 24, while the maximum is 312. The binomial (13), Korsuň 1 (8) and Prodan (7) functions had the lowest rank total (57–67). The hyperbola (10) proved to be the worst function, having the greatest rank total (303).

Figures 1 and 2 present examples of height curves plotted for the Korsuň, Prodan and hyperbolic functions. In the case of spruce in the Chełmsko control unit (Fig. 1) the hyperbola provided negative height values for the smallest diameters at breast height. The Korsuň curve, after the maximum value was exceeded, started to fall, but it was more marked only after the range of measurement data was exceeded. The Prodan curve turned out to be accurate over the entire range of values for diameters at breast height. In the case of beech in the Trzebieszowice control unit plot (Fig. 2) the Korsuň and Prodan curves are generally identical, while the range of the hyperbola diverges considerably, particularly at high values of diameter at breast height.

The most useful and suitable model was Prodan function (7). The empirical parameters obtained for this function are presented in Table 10. Values of these parameters for both methods of calculation are usually similar, but they can differ from each other in terms of sign, like in case of silver fir in the Chełmsko control unit.

DISCUSSION

An interesting result of these comparisons was connected with the observed differences in values of empirical parameters for the functions obtained using analytical and iterative methods. The iterative method proved to be superior in this respect. The analytical method was not better in any of the comparisons.

In even-aged forests, in which the structure of diameter at breast height is close to the normal distribution (i.e. symmetrical), it seems easy to select a model

Table 8. Coefficients of determination R^2 and coefficients of random variation W_e measuring goodness of fit for function graphs to empirical data for silver fir in the Chełmsko control unit

No.	Provided by the analytical method		Provided by the iterative method	
	R^2	W_e	R^2	W_e
1	0.8270	8.95	0.8278	8.94
2	0.8225	9.04	0.8240	9.04
3	0.8178	9.17	0.8228	9.07
4	0.8273	8.94	0.8280	8.94
5	0.8185	9.13	0.8209	9.12
6	0.8040	9.45	0.9067	9.37
7	0.8273	9.01	0.8309	8.86
8	0.8315	8.94	0.8331	8.80
9	0.8164	9.23	0.8164	9.23
10	0.8116	9.35	0.8116	9.35
11	0.8202	9.13	0.8202	9.13
12	0.8266	8.96	0.8275	8.95
13	0.8346	8.87	0.8346	8.76

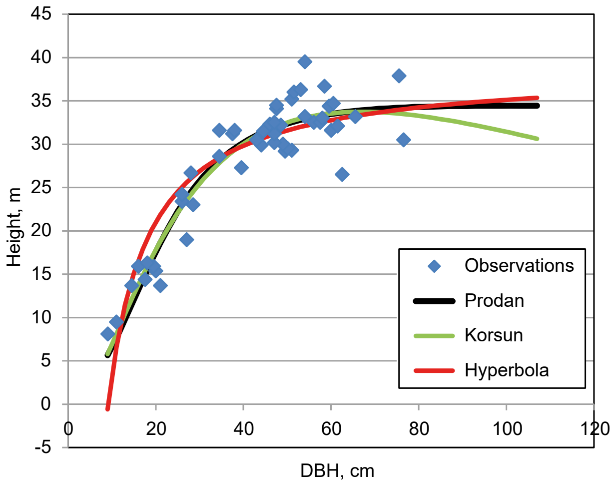

Fig. 1. Height curves for Norway spruce in the Chełmsko control unit

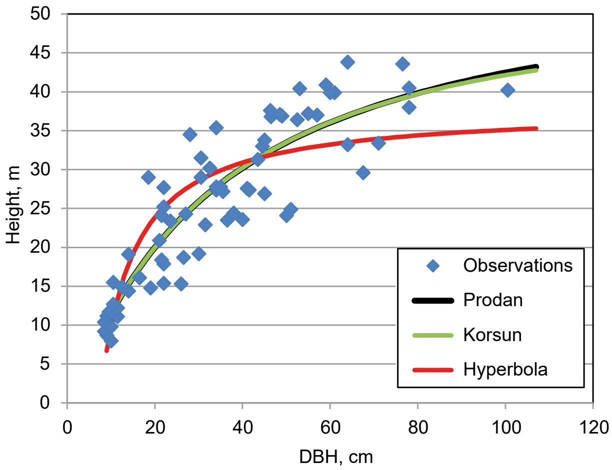

Fig. 2. Height curves for European beech in the Trzebieszowice control unit

Table 9. Totals of goodness of fit ranks provided by functions depending on the coefficient of determination R^2 and coefficient of variation of reidual W_e

No.	Equation	Analytical method		Iterative method		Total of all ranks
		R^2	W_e	R^2	W_e	
1	$H = ae^{-k/D} + 1.3$	51	47	46	46	190
2	$H = \left(\frac{D}{a+bD}\right)^3 + 1.3$	38	38	45	44	165
3	$H = \left(\frac{1}{a+bD}\right)^{2.5} + 1.3$	58	58	43	42	201
4	$H = e^{a+b/D}$	50	49	49	47	195
5	$H = \left(\frac{D}{a+bD}\right)^2 + 1.3$	40	36	40	39	155
6	$H = a\left(\frac{D}{b+D}\right)$	54	47	38	50	189
7	$H = \frac{D^2}{a+bD+cD^2} + 1.3$	20	20	14	13	67
8	$H = e^{a+b\ln D+c\ln 2D}$	16	20	13	12	61
9	$H = a + b\ln D$	45	46	53	52	196
10	$H = a + b\frac{1}{D}$	75	75	77	76	303
11	$H = a\sqrt{\log(D+1)} + b$	43	47	57	56	203
12	$H = a\left(\frac{D}{1+D}\right)^b + 1.3$	47	48	54	53	202
13	$H = a + bD + cD^2$	9	15	17	16	57

for the height curve. Postulates of dendrometry in relation to these curves are as follows: (Michailoff, 1943): 1 – for $D = 0$ it should be $H = 1.3$; 2 – for $D = \infty$ it should be $H =$ const., i.e. there should be an upper asymptote, for $D = m$ (in the point of inflexion) it should be $H'' = 0$.

Age variation and the structure of diameter at breast height in selection forests produce another problem. In uneven-aged stands the distribution of diameter at breast height is right-skewed – thin trees predominate in number and they comprise most of the sample population. As a result the distribution of measured heights is also right-skewed. This means that the course of the curve (i.e. parameter values) is more strongly affected by these trees, which dimensions have a minimal effect on volume, which determination is the ultimate objective of the study. For this reason height curves should possibly have the best goodness of fit to observations in the class of the thickest trees, determining volume and increment in volume.

According to Przybylska and Przybylski (1994), the non-uniform distribution of the independent variable (diameter at breast height) is one of the causes for the more accurate estimation of equation parameters using iterative methods. Another cause is connected with transformations of data, aiming at linearisation of functions, resulting in equalisation errors.

Table 10. Empirical coefficients of Prodan function ($H = D^2/(a + bD + cD^2) + 1.3$) determined according to analytical (an) and iterative (it) method

Control unit	Species	Method	Parameters		
			a	*b*	*c*
Trz	spruce	an	26.057318202	–0.523923593	0.028185713
		it	21.010516483	–0.338961409	0.026342628
Trz	beech	an	3.489929192	0.563032540	0.019039134
		it	1.750348503	0.626065667	0.017860604
Trz	sycamore maple	an	–1.148894380	0.867383357	0.017341214
		it	–3.487660558	1.062605620	0.013614456
Ch	spruce	an	19.132764333	–0.438356250	0.033582472
		it	19.463164559	–0.384204337	0.032067393
Ch	beech	an	8.457345086	0.601176564	0.022113823
		it	6.482127830	0.745325285	0.019598506
Ch	fir	an	21.179229533	–0.173544796	0.029737079
		it	15.866245521	0.063523372	0.026979060

Trz – Trzebieszowice, Ch – Chełmsko.

The best goodness of fit to empirical data was obtained using the equations of the 2nd degree parabola (13), the Prodan function (7) and the Korsuň function (8). The parabolic equation may be used only conditionally, as it does not meet the postulates imposed on height curves by dendrometry. However, in many cases a very good fit is obtained (Bruchwald, 1970; Meixner, 1964; Näslund, 1929), similarly as it was the case in this study. For this reason this function does not have to be rejected; nevertheless, it should always be verified what results are obtained for extreme values of diameter at breast height. If there are negative height values or the maximum value of the function was exceeded and the approximated height decreases with an increase in diameter at breast height, the function has to be rejected. The Korsuň function does not meet the first postulate and its application has to follow similar reservations as those for the application of the 2nd degree parabola. The Prodan function seems to be the function, which may be applied with no reservations also in automated computations.

The analysis of the other functions shows that the hyperbolic function may be easily discarded. Its only advantage results from the simplicity of calculations; however, in view of the common use of electronic computation technology this advantage is no longer important.

CONCLUSIONS

1. Determination of empirical coefficients of equations describing the dependence between diameter at breast height and tree height using the iterative method is more accurate than that provided by the analytical method, transforming equation to a linear or polynomial form.
2. In uneven-aged stands with a positive (right-skewed) distribution of diameter at breast height the Prodan function (7) has the largest number of advantages, including accurate fit to data, comparable to that of the 2nd degree polynomial function (13) or the Korsuň function (8).

3. The other functions, including classical functions, meeting postulates of dendrometry (i.e. Michailoff (1), Näslund (5)), do not show such a good fit.
4. An unsatisfactory goodness of fit to data was found for the hyperbolic function (10) and as such it may be rejected.

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OCENA FUNKCJI OPISUJĄCYCH ZALEŻNOŚĆ PIERŚNICY I WYSOKOŚCI DRZEW W DRZEWOSTANACH Z PRZEMIANĄ SPOSOBU ZAGOSPODAROWANIA LASU NA PRZERĘBOWY W SUDETACH

ABSTRAKT

Na potrzeby określania miąższości drzewostanów ze zmianą sposobu zagospodarowania lasu w Sudetach przetestowano 13 funkcji opisujących zależność pomiędzy pierśnicą a wysokością drzew. Pomiary drzew wykonano w dwóch jednostkach kontrolnych w 2013 i 2015 roku. Testowanie funkcji wykonano na świerku, buku, jaworze i jodle. Parametry funkcji określano metodą przekształcenia równań do postaci liniowej lub wielomianowej i rozwiązania układu równań normalnych, oraz metodą iteracyjną Levenberga-Marquardta, zastosowaną w programie Statistica 12. Dopasowanie wykresów funkcji do danych było lepsze z wykorzystaniem metody iteracyjnej. Najlepszym dopasowaniem charakteryzował się wykres paraboli 2. rzędu. Dobre dopasowanie uzyskano z zastosowaniem funkcji Korsunia (1935) ($H = \exp(a + b \cdot \ln(D) + c \cdot \ln^2(D))$ oraz funkcji Prodana (1951) ($H - 1.3 = D^2/(a + b \cdot D + c \cdot D^2)$). Dopasowanie pozostałych testowanych funkcji było wyraźnie mniejsze. Mimo bardzo dobrego dopasowania, wykresy paraboli 2. rzędu i funkcji Korsunia nie spełniają wszystkich postulatów dendrometrii stawianych krzywym wysokości. Do dalszego praktycznego stosowania wybrano więc funkcję Prodana.

Słowa kluczowe: krzywe wysokości, aproksymacja, metoda iteracyjna