

LIMITATIONS OF RIPLEY'S $K(t)$ FUNCTION USE IN THE ANALYSIS OF SPATIAL PATTERNS OF TREE STANDS WITH HETEROGENEOUS STRUCTURE

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Abstract. The use of Ripley's $K(t)$ function for analysis of spatial structure of tree stands has recently become easier due to the growing availability of advanced software. Unfortunately not every data set is suitable for analysis with classical estimator of $K(t)$ function. Large directional differences in density of trees in analysed region cause that estimators suggest aggregated distribution of trees even if it is not true. This paper presents the review of proposals to solve this problem based on dividing of the analysed region into homogeneous sub-regions. Analyses carried out on separate sub-regions can furnish viable information about spatial structure of tree stand. Besides, examples of such analyses made with commonly available non commercial software are presented.

Key words: Ripley's $K(t)$ function, tree spatial distribution, heterogeneous pattern

INTRODUCTION

The Ripley function [1977] has recently become a popular analytic tool used for the analyses of spatial structure of forest stands. In Poland, this method has been given a growing interest since the work of Szwagrzyk and Ptak [1991]. The actually still growing availability of software for the spatial analyses justifies the assumption that also in the future the method will continue to be popular, even more than nowadays. Weiner [1985] has presented very convincingly the threats resulting from the ease of conducting calculations without the awareness of the statistical basis of the correlation and regression calculi. Similar is the situation with the analyses of point data as carried out with use of the Ripley function. The proper application of the function needs to accept several assumptions that not necessarily are fully met in reality. The aim of this paper is to give a more popular presentation of selected terms from the field of spatial data analysis, such that have to be understood if one decides to expose the data for the analysis

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using the Ripley's function. Another goal is to suggest adequate methods for overcoming of some constraints of application of the function for the analysis of heterogeneous point patterns.

STOCHASTIC PROCESS

In the scientific papers dealing with Ripley's function, application for the analyses of spatial distribution of individuals of plant or animal populations very often are present statements expressed in terms of spatial stochastic processes. The presented in this paper description of a spatial stochastic process and its properties is only limited to a plain, and the presentation itself is rather of illustrative character, less formal. A more formalized and comprehensive description of the problem may be found in the handbooks dealing with spatial data analysis [Diggle 2003, Cressie 1993, Stoyan et al. 1987].

One of the most frequently addressed aspects of the spatial structure of forest stands has been the type of point pattern formed by trees or, more precisely, by tree stems distributed on the area of stand under study. Usually, the information concerning the size of tree stems is omitted and each tree is represented in the form of a point of given perpendicular or angular coordinates values. As a result, the actual object of study is the set of coordinates describing the distribution of points over the plain (further in text referred to as point pattern). In order to univocally identify such points, they will be named "objects" further in the text.

One may imagine the spatial stochastic process as a mechanism generating the point patterns over the entire plain. A specific point pattern is called the realisation of the stochastic process. Despite the fact that the number of realizations of a stochastic process is unlimited, every individual point pattern generated by given process bears its sign. The analysis of spatial relationships between objects allows for the formulation of hypotheses concerning the essence of the process that have generated them. A majority of analytical methods dealing with point data (including also the Ripley's function) are based on the formally (mathematically) described properties of the stochastic processes. An user of a statistical software package might not be aware of the fact that for the proper use of the methods an assumption must be fulfilled that the point pattern of objects as observed on the research plot is fragment of a realization of unknown stochastic process. The assumption itself has no limitations concerning the type of point pattern formed by objects on the research plot. Regardless of the fact that the given point pattern has been generated by either a process producing random pattern, or regular or cluster distributions, the analyzed point pattern needs to be a realization of a process producing the **stationary** and **isotropic** patterns. If the last mentioned conditions are not met, the output concerning the type of point pattern gained with use of Ripley's function may be incorrect. The analog to this situation would be incorrect use of t-Student's test to compare means of two samples in case that distributions of values in those samples are definitely not normal. Further in this paper a detailed description will be given of an effect with lack of stationarity (homogeneity) of point pattern for the results of the Ripley's function application.

POISSON PROCESS – AN EXAMPLE OF A STATIONARY SPATIAL PROCESS

When studying the type of trees distribution, the null hypothesis is most often put forward that the actual point pattern is random. The researchers are particularly interested in rejecting the hypothesis, because the non random point pattern may give evidence for the activity of factor(s) influential for the spatial structure of a stand and thus – giving hints for further discussion of the results.

Very many methods of spatial structure analysis of population are based, either directly, or indirectly, on the homogeneous Poisson point process as the model of random distribution of objects. Comparing the properties of a given point pattern with those of Poisson process (or, more precisely: with the properties of the Poisson process's generated point patterns) one may verify the null hypothesis of the concordance of given point pattern with the realizations of Poisson spatial process. An important parameter describing a stochastic process is intensity. The first order intensity λ means the average number of objects per area unit ($|ds|$) around any point s [after Bailey and Gatrell 1995, page 77] and may be defined with use of formula 1:

$$\lambda(s) = \lim_{|ds| \rightarrow 0} \left\{ \frac{E(Y(ds))}{|ds|} \right\} \quad (1)$$

where: ds – a small region around point s ,
 $|ds|$ – is the area of the region,
 $Y(ds)$ – the number of objects in the region.

If a stochastic process is stationary, then all probabilistic statements regarding the process for any selected observation region A are invariable in case of any shift of the region over the entire plain (entire realization). For instance, the mean (expected) number of objects generated by the homogenous Poisson process in study region A of area equal $|A|$ is the two element product $\lambda \cdot |A|$, regardless the actual location of study region over the whole particular realization of Poisson spatial process. The stationary process is sometimes also called the homogeneous process, analogically the non-stationary processes are also called heterogeneous, or inhomogeneous.

The fact that the expected number of objects per areal unit is constant in every spot of the plain in the Poisson process does not imply that in given sample plot of size $|A|$ the number of present objects will always be the same and equal to $n = \lambda_1 \cdot |A|$. The number of objects n that may occur in a sample plot of size $|A|$ is a variable undergoing the Poisson distribution (formula 2).

$$\Pr(n) = e^{-\lambda|A|} \frac{(\lambda|A|)^n}{n!} \quad (2)$$

where: e – base of the natural logarithm.

The graphical illustration of the statements' correctness is presented in Figures 1 a-d. The map shown in Figure 1 a presents a fragment of realization of spatial Poisson process. The fragment of dimension 100 by 100 meters, containing 400 objects referred to further in the text as the study region was obtained using a computer simulation. The first order intensity for this particular point pattern may be assessed equal 0.04 objects

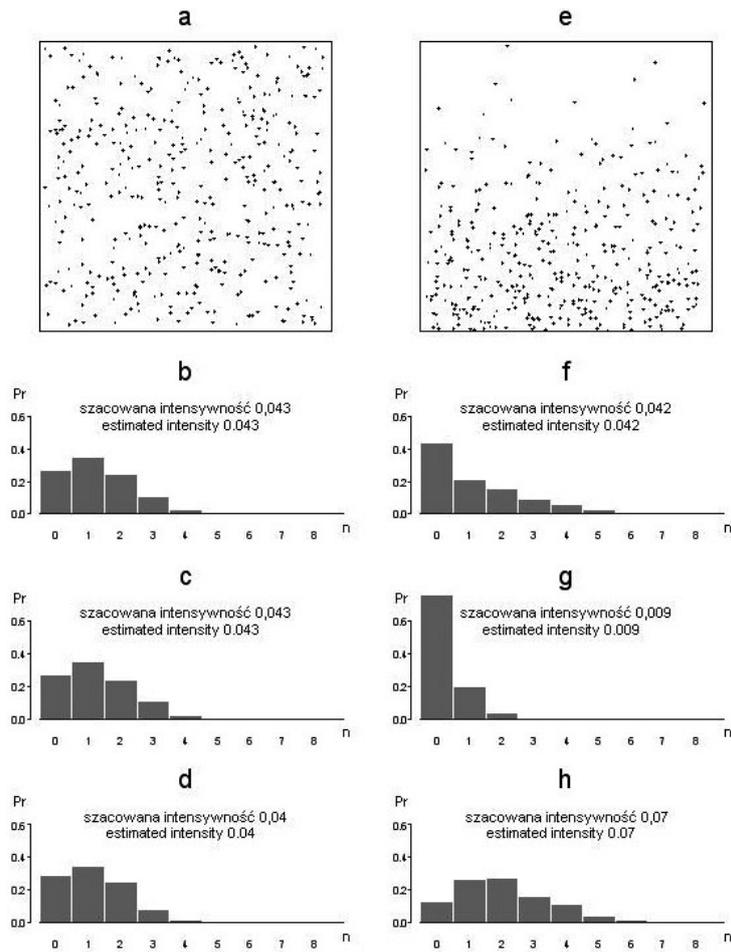


Fig. 1. Distributions of number of objects found on sample areas in homogeneous (b, c, d) and heterogeneous (f, g, h) point patterns located on whole area of point pattern (b, f), only in upper half (c, g) and lower half (d, h)

Rys. 1. Rozkład liczby obiektów na powierzchniach próbnych w rozmieszczeniu jednorodnym (b, c, d) i niejednorodnym (f, g, h) umieszczanych na całej powierzchni rozmieszczenia (b, f) oraz w górnej (c, g) i dolnej połowie (d, h)

per m^2 . Under the map three histograms are included, describing the probability distribution of finding specific number of objects in a small sized sample area from the presented random point pattern (Figs. 1 b, c and d). They are of the empirical character, because they have been determined on the basis of counting objects in sample plots of area $|A| = 30 m^2$. The computer algorithm has produced as many as 1000 circular sample plots of radius $r = 3.09 m$ in the study region. The objects number were summed only in those sample plots that where placed entirely within the accepted study region.

The algorithm had repeated random placement of the center of the sample plot as long as 1000 such samplings were collected. In the figures there is also available information concerning the average number of objects per sample plot (estimated intensity). Regardless the accepted procedure of sampling: if sample plots were collected from the entire investigated point pattern area (Fig. 1 b), or from its northern half (Fig. 1 c), or southern half (Fig. 1 d), the obtained histograms are nearly identical and the mean number of objects found in sample plots is also very similar in every case. The above is a graphical illustration of homogeneity of the studied process.

For the comparison, an example of heterogeneous processes realization was shown in Figure 1 e. Also this study region has dimension 100 by 100 m and it contains 400 objects. In this case, the empirical distributions of finding n objects in a circular sample plot as determined for the entire distribution (Fig. 1 f) and for its northern half (Fig. 1 g) and southern half (Fig. 1 h) are clearly different. The obviously visible differences in the shape of the distributions have been due to equally easily visible difference in local intensity of objects (first order intensity) between the northern and southern parts of the distribution.

Bailey and Gatrell [1995] define the effect of differentiated density of objects in the study area for the results of study on point pattern peculiarities as the first order effect. The terms "not stationary" or "heterogeneous" as applied for the point patterns suggest that the local values of objects density (first order intensity) are varying firmly over the entire area of the point pattern. The heterogeneity is usually fact concerning the density variability as observed in spatial scales definitely larger than the average distance between particular objects.

ANALYSING HETEROGENEOUS DISTRIBUTIONS PATTERNS THROUGH DISCRIMINATION OF HOMOGENEOUS FRAGMENTS

The most often used estimator of the $K(t)$ function has been worked out for the homogeneous point patterns (stationary and isotropic). According to Ripley [1977], in order to reduce the edge effect a coefficient correction has to be applied (formula 3).

$$\hat{K}(t) = \sum_{i \neq j} \sum w_{ij}^{-1} I(u_{ij}) / \lambda^2 |A| = \frac{|A|}{n^2} \sum_{i \neq j} \sum w_{ij}^{-1} I(u_{ij}) \quad (3)$$

where: u_{ij} means the distance between objects i and j ,

$$I_t = \begin{cases} 1, & \text{for } u_{ij} \leq t \\ 0, & \text{for } u_{ij} > t \end{cases}$$

w_{ij} – correction coefficient applied for the reduction of the edge effect,
 $|A|$ – the area of study region.

Every statistical program that uses such an estimator, is capable of conducting calculations for the heterogeneous point pattern (eg, for the distribution shown in Figure 2 a, with the square study region of 1×1 m size, and the number of objects = 500). Because of the first order effect, their result is usually useless. In case significant directional differences occur in the value of local density of objects, within the region of study, the shape of transformed estimator of the Ripley function ($\hat{L}(t) = \sqrt{\hat{K}(t) / \pi - t}$) is often of

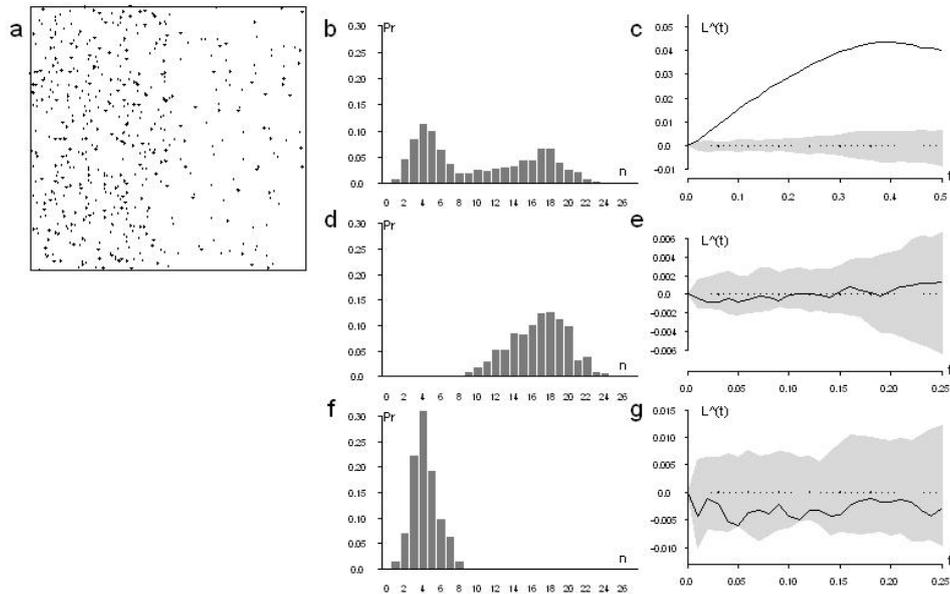


Fig. 2. Illustrative analysis of heterogeneous 1×1 m square point pattern: the map of point pattern (a), distribution of number of objects found on circular sample area with radius 0.08 m located on whole area of point pattern (b), only on its left (d) and right (f) half, estimator of Ripley's $L^*(t)$ function with confidence limits ($\alpha = 0.05$) for whole pattern (c) and for its left (e) and right (g) part

Rys. 2. Przykładowa analiza kwadratowego fragmentu (1×1 m) rozmieszczenia niejednorodnego: mapa rozmieszczenia (a), rozkład liczby obiektów znajdujących na kołowej powierzchni próbnej o promieniu 0,08 m w całym rozmieszczeniu (b), w jego lewej (d) i prawej połowie (f) oraz przebieg estymatora funkcji Ripleya $L^*(t)$ na tle przedziałów ufności ($\alpha = 0,05$) dla całości rozmieszczenia (c), jego lewej (e) i prawej połowy (g)

the ascending character in the whole range of analyzed spatial scales (Fig. 2 c) suggesting thus the studied point pattern is clustered. Figure 2 c illustrates the effect of density variability of objects as observed at a large scale, for the results of analysis made with use of Ripley function of spatial relations between objects at a smaller scale. Bailey and Gatrell [1995, page 91] call this phenomenon the influence of first order effect on the second order effect. They suggest that given a strong first order effect in a studied point pattern, a proper solution would be to analyze the spatial relations between objects after dividing the entire point pattern into smaller fragments and considering each such fragment independently from the others, assuming the homogeneity of point pattern in the smaller fragments. The frequency distribution of objects in 1000 of circular sample plots of radius equal 0.08 m and established within the studied point pattern (Fig. 2 b) has the bimodal shape. Such a distribution pattern suggests that the point pattern under study consists of at least two different parts of different densities. The two peaks visible in the distribution made for entire point pattern (Fig. 2 b) correspond with the peaks observed in the distributions constructed in the same way for the left hand and the right hand half of the studied point pattern (Figs. 2 d and f).

It can be assumed that the homogenous fragments as suggested by Bailey and Gatrell are equivalents of both the left and right halves of the studied point pattern. The course of the Ripley function's estimator both for the left half and right half of the point pattern under study (Figs. 2 e and g) suggests the random pattern of the objects. It can be therefore stated that the removal of the first order effect (directional differentiation of density) has allowed for the determination of real spatial relations between the objects of the studied point pattern.

Frequent ecological observations show that the heterogeneous spatial distribution pattern is a relatively common phenomenon. In terms of practice, as homogeneous may be accepted such a point pattern that is characteristic of the similar (not necessarily identical) density of individuals per area unit all over the study region. The visible during the eye inspection of the distribution map of objects directional changes in density in the study region should force one to reject such an assumption. An additional hint may be given by the specific course of the $\hat{L}(t)$ estimator. If the above mentioned curve is of ascending character over nearly the entire scope of the analysed spatial scales and it peaks (suggesting thus the cluster pattern of objects) for t value close to t_{\max} , it is likely that the objects are arranged following the heterogeneous pattern rather than the homogenous one. While applying the coefficient correction of the edge effect for the estimator $\hat{L}(t)$ the largest spatial scale analyzed t_{\max} shall not be larger than half of the shorter rectangular side length of the area of study region; given an irregular shape of study region, t_{\max} should not exceed a third of the longest linear dimension of the region [Rowlingson and Diggle 1993]. These constraints are direct consequences of the formula accepted and applied for the correction of the edge effect. There is another, even more important in this case, rationale. The graph in Figure 2 c suggests that the objects in 1×1 m square (Fig. 2 a) are distributed following the cluster pattern. In case of homogeneous point patterns, the spatial scale t , for which the $\hat{L}(t)$ estimator reaches its statistically significant maximum value, determines the mean distance between objects in clusters, which is interpreted in terms of the radius of clusters [Rebertus et al. 1989]. Interpreting this way the data from Figure 2 c, one could come to the conclusion that the objects in the analyzed point pattern are distributed in clusters of 0.8 m diameter which is unlikely for such small study region. In practice, this could mean that the study region contains only one large cluster of individuals. The visual assessment of Figure 2 distribution may suggest that the studied region has included partly a fragment of a larger cluster located to the left, but the actual size of the study region gives no hints that would be helpful in the assessment of the size of this cluster.

DETERMINATION OF HOMOGENEOUS SUBREGIONS – EXTERNAL FACTORS

According to a commonly expressed belief, the distribution variability of individuals as observed in a large spatial scale, is conditioned by external factors such as climate or soil. On the other hand, however, the pattern of distribution at smaller spatial scales is conditioned rather by the external factors, the latter resulting from the mutual relationships between particular individuals such as competition or specific properties of studied species, like eg, vegetative reproduction [Pélissier and Goreaud 2001]. Such an attitude may suggest the selection of a method of solving the problems connected with

the analysis of heterogeneous point patterns such that would be based on the division of the study regions into subregions homogeneous in regard with the external factor, the latter may affect the distribution of trees [Goreaud and Pélissier 1999, Pélissier and Goreaud 2001]. Figure 3 shows a fragment of trees distribution (sized 35×67.5 m) as observed on a research transect in the Białowieża National Park [Bernadzki et al. 1998], and located on the borderline between the podzol and peat soils. The continuously ascending estimator $\hat{L}(t)$ calculated for the entire of the distribution (Fig. 3 b) has the characteristic signs suggesting the heterogeneous character of the studied point pattern. The distribution of trees growing in the podzol soil (Fig. 3 c) is random while the graph describing the distribution of trees in the peat soil area (Fig. 3 d) suggests the occurrence of small scale (up to about 2.5 m) clusters. The curve of the Ripley function's estimator for this part of the distribution is placed for the larger spatial scales close to the upper confidence limit range, and for the scales of 7 m and 10 m it goes slightly above the limit.

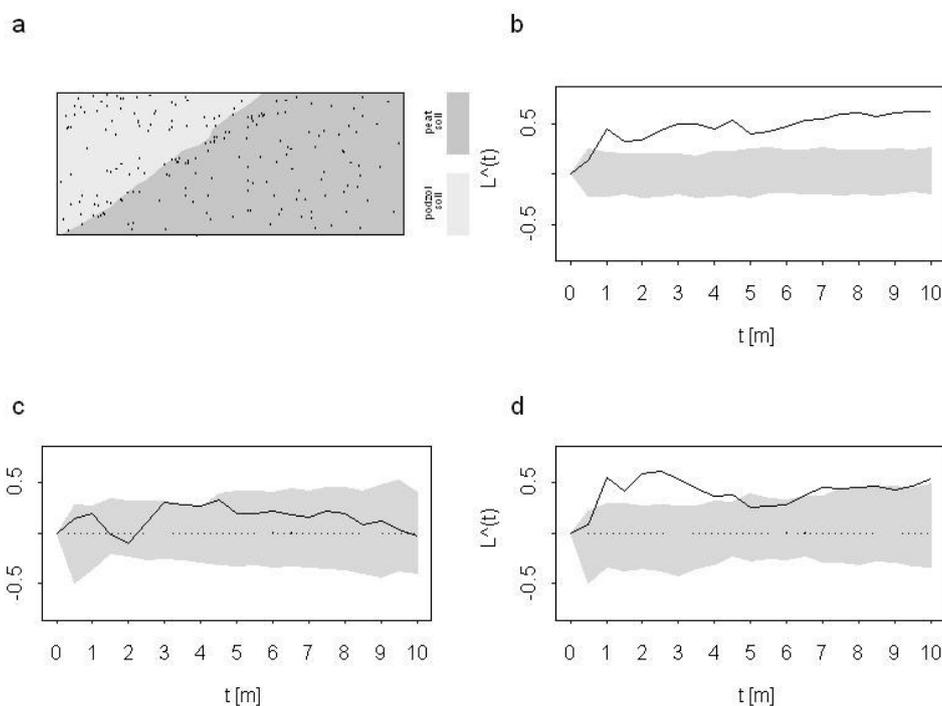


Fig. 3. Analysis of tree spatial pattern on research plot in the Białowieża National Park: trees and soil types map on research plot, estimator of Ripley's $L^{\wedge}(t)$ function with confidence limits ($\alpha = 0.05$) for whole plot (b), for trees on podzol soil (c) and on peat soil (d)

Rys. 3. Analiza rozmieszczenia drzew na powierzchni badawczej w Białowieżskim Parku Narodowym: rozmieszczenie drzew i mapa zalegania różnych typów gleb na powierzchni badawczej (a), przebieg estymatora funkcji Ripleya $L^{\wedge}(t)$ na tle przedziałów ufności ($\alpha = 0,05$) dla całości rozmieszczenia (b), dla drzew rosnących na glebie bielcowej (c) i na glebie torfowej (d)

Close to the border with the podzol soil the density of trees is visibly higher than in the rest of the area covered by peat soil; it is therefore not reliable to assume that this subregion is homogeneous and the observed shape of estimator curve cannot be accepted as well. In the case of the example presented above, the method of delimitation of fragments homogeneous with respect to the effect of external factor activity has been justified only partly.

IDENTIFICATION OF HOMOGENEOUS SUBREGIONS – FIRST ORDER INTENSITY

If in the region of study the heterogeneity of point pattern is visible but no external factor that would be responsible for the situation can be identified, or there is lack of data concerning the possible external factor, it is still possible to identify homogeneous regions. The rationale for the determination is supplied by the map of local densities of objects per unit of area [Pélissier and Goreaud 2001]. The delimitation between particular homogeneous subregions takes part in the vicinity of an isoline showing a chosen level of local density of objects. The selection of the critical level λ_k is achieved based on an analysis of the empirical probability distribution of finding determined number of objects in small sample plots ISI distributed in the study region (similar distributions are presented in Figs. 1 and 2). Figure 4 a presents the results of summing the objects in circular sample plots of radius equal 7.5 m and located in the point pattern from Figure 3 a. The computer algorithm [Goreaud and Pélissier 2001] has placed 9600 sample plots in the region of study so that their central points were arranged following the perpendicular grid spaced 0.5625×0.4375 m. If the sample plot was placed on the border of study region, the number of objects was increased by means of the coefficient correction, similarly as in formula 3. The histogram presenting the frequency of objects in sample plots is of bimodal character (having two local maxima). As the class discriminating the two parts of the histogram, which is a local minimum between the two peaks of the distribution, the class containing 17 objects has been selected. In order to obtain the approximate value of the local density of objects discriminating the two homogeneous areas, λ_k (9.62 pcs./100 m²), it was necessary to divide the number of objects in the suggested class ($n_k = 17$) by the area of the sample plot $ISI = 176.7146$ m². Figure 4 b presents the local densities of objects in the study region, and two isolines of local density 9.62 pcs./100 m². These isolines delimit the area of elevated trees density (marked with the lighter color), to a large degree identical with the border zone between the mineral and the organic soil in the study region, and the areas of smaller density (the latter marked with darker color). The analysis of data from the areas characterized by the smaller density (Fig. 4 c) shows for the random distribution pattern of trees growing there. The trees growing in the region of elevated density have established clusters visible in the spatial scales 1 m and 3 m (Fig. 4 d). In case of this method of homogeneous areas discrimination, the estimator curve shows in none of the identified parts of distribution the tendency to grow with the spatial scale.

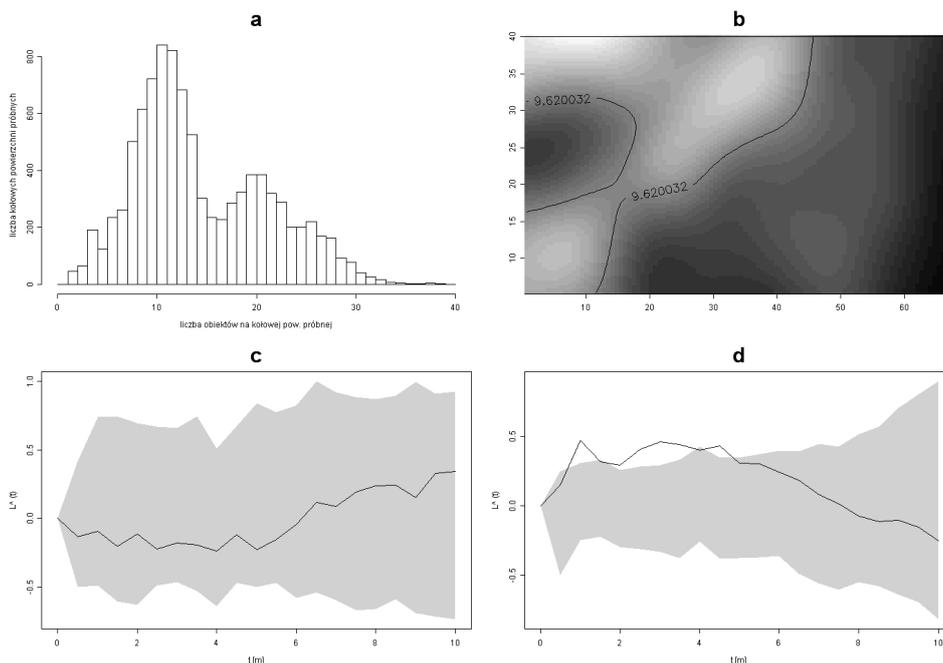


Fig. 4. The analysis of tree spatial pattern on research plot in the Białowieża National Park: distribution of number of trees found on circular sample area with radius 7.5 m located on research plot (a), map of local tree density (b), estimator of Ripley's $L^*(t)$ function with confidence limits ($\alpha = 0.05$) for parts of pattern with local density below 9.62 trees·100 m⁻² (c) and for the rest of the pattern (d)

Rys. 4. Analiza rozmieszczenia drzew na powierzchni badawczej w Białowieżskim Parku Narodowym: rozkład liczby drzew odnajdowanych na kołowych powierzchniach próbnych o promieniu 7,5 m umieszczanych w badanym rozmieszczeniu (a), mapa wartości lokalnego zagęszczenia drzew (b), przebieg estymatora funkcji Ripleya $L^*(t)$ na tle przedziałów ufności ($\alpha = 0,05$) dla części rozmieszczenia o zagęszczeniu drzew poniżej 9,62 szt·100 m⁻² (c) i dla pozostałej części rozmieszczenia (d)

METHODICAL CONSTRAINTS

To practically implement the idea of homogenous subregions discrimination the application of procedures is needed such that would allow for the calculation of function K estimator values for the study regions characteristic of irregular shapes. One of the very earliest computer applications facilitating this task's accomplishment has been that by Rowlingson and Diggle [1993]. A big challenge for a programist is how to implement the mathematical formulae necessary for the calculation of the correction coefficient values (w_{ij} in formula 3). Similarly as the earlier cited researchers, also Goreaud and Pélissier [1999] had assumed that a homogenous subregion may be represented by an irregular polygon. Although in the perpendicular study region to properly calculate the correction coefficient of the edge effect only 3 formulas are needed [Haase 1995], in the methods proposed by Goreaud and Pélissier [1999] as many as 8 different formulas

have to be used. The Goreaud and Pélissier [2001] algorithm seems to be more flexible than that of Rowlingson and Diggle [1993], because it enables conducting an analysis in the study region after exclusion of a smaller polygon within the region. In the analysis presented in Figure 4 c the algorithm worked out by Baddeley and Turner [2005] was applied, allowing for the simultaneous analysis of distribution of trees in two spatially separate fragments of the study region.

Wiegand and Moloney [2004] emphasize the impact the coefficient correction of the border effect may have for the results of heterogenous point pattern analyses. Figure 5 a shows a point pattern in which the objects are concentrated along the borderlines of the study region. The course of the Ripley function curve for this distribution (Fig. 5 c) suggests the presence of a very strong clusterness at the scale of 20 m, and a very strong regularity in the scale of 50 m. Although the above mentioned clusterness has undoubtedly

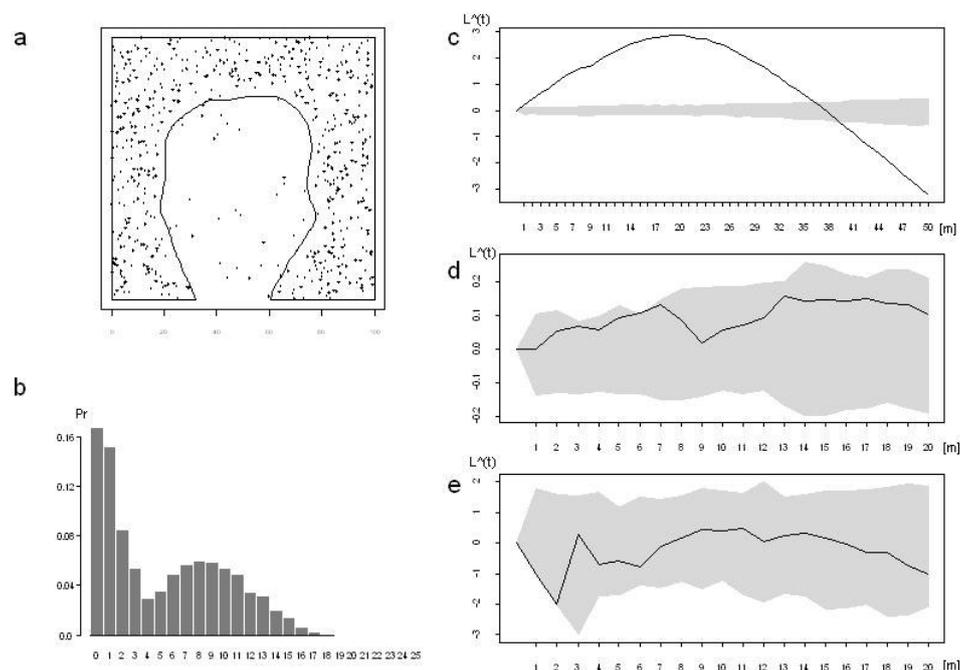


Fig. 5. The example of influence of first order effect and factorial edge effect correction on estimator of Ripley's function: tree pattern with line drawn between parts with apparently different tree densities (a), distribution of number of trees found on circular sample area located on research plot (b), estimator of Ripley's $L^{\wedge}(t)$ function with confidence limits ($\alpha = 0.05$) for whole plot (c), for part of pattern with higher tree density (d) and with lower tree density (e)

Rys. 5. Przykład wpływu efektu pierwszorzędowego oraz współczynnika korekty efektu brzegowego na przebieg estymatora funkcji Ripleya: rozmieszczenie z obiektami skoncentrowanymi przy granicach wraz z linią oddzielającą obszary o zdecydowanie różnym zagęszczeniu drzew (a), rozkład liczby drzew odnajdowanych na kołowych powierzchniach próbnych umieszczonych w badanym rozmieszczeniu (b), przebieg estymatora funkcji Ripleya $L^{\wedge}(t)$ na tle przedziałów ufności ($\alpha = 0,05$) dla całości rozmieszczenia (c), dla części z większym zagęszczeniem drzew (d) i z mniejszym (e)

been result of the first order attributes effect of the point pattern under study, the regularity of pattern has been possible to be detected thanks the application of the coefficient correction of the edge effect. For the majority of objects, the number of objects present in their vicinity decreases with the growing diameter of the analyzed vicinity, especially above 20 m (the vicinity includes then the area within the study region characteristic of small density of objects). This results in a rapid fall of the estimator curve, down the level below the lower confidence limit for the spatial scales exceeding 38 m. The obeying of the principle of determination of homogeneous areas based on a local value of objects density makes it possible to reveal the true nature of the studied point pattern. The threshold value of density (4 pcs./100 m²) between the homogeneous regions was determined with use of circular sample plots sized 100 m² each (Fig. 5 b). The areas were delimited on the basis of the course of proper density isoline (Fig. 5 a) obtained similarly as in Figure 4. Both in the area of elevated and the area of lowered density of objects, their distribution follows the random pattern (Figs. 5 d and e). All calculations were made in the environment of R software (R Development Core Team 2005).

Even though the above quoted solutions have participated to the substantial progress in the analysis of heterogeneous point pattern, they also have frequent limitations. The best results of determination of homogeneous regions are usually obtained in case the two identified subregions are characteristic of the random point pattern of objects [Pélissier and Goreaud 2001, Wiegand and Moloney 2004]. Moreover, the results of the analysis may, to a large extent, be dependent on the number, size and location of circle sampling plots (as generated by the computer), but also, on the applied method of drawing the border isoline of local densities of the objects. Besides, also the very substantial question of criteria of decision whether the studied distribution needs to be accepted as homogenous or not, is still poorly worked out.

This method of analysing heterogeneous point pattern has a great potential. It is conditioned however by the necessity to select the homogeneous fragments of point pattern on the basis of objective and explicitly formulated criteria. The relative ease of division of analyzed point pattern into smaller fragments may, given arbitrary selection, lead to very differentiated results. If the researcher is not able to justify the accepted division criteria of the study region, then even statistically significant results, will be useless from the point of view of the natural sciences. As a conclusion the following citation may be put forward: "Although part of the art of spatial point process analysis is trying to disentangle these two effects (ie, the first order effect, and the second order one – author's comment) in order to better understand the process generating the observed events, inferring process form pattern is ultimately a question of judgement and may not always be clear cut" [Gatrell et al. 1996].

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OGRANICZENIA W STOSOWANIU FUNKCJI RIPLEYA DO BADANIA STRUKTURY PRZESTRZENNEJ DRZEWOSTANÓW ROZMIESZCZENIA NIEJEDNORODNE

Streszczenie. Dzięki dużej dostępności odpowiedniego oprogramowania coraz łatwiejsze staje się zastosowanie funkcji Ripleya $K(t)$ do analizy danych o rozmieszczeniu pni drzew w drzewostanie. Niestety nie każdy zestaw danych nadaje się do analizy za pomocą klasycznego estymatora funkcji $K(t)$. Duże kierunkowe zmiany zagęszczenia drzew na badanej powierzchni powodują, że estymator może wskazywać skupiskowość rozmieszczenia drzew w sytuacji, gdy ona nie istnieje. W pracy przedstawiono przegląd literatury dotyczącej rozwiązania tego problemu za pomocą podziału badanej powierzchni na mniejsze powierzchnie jednorodne. Analizy rozmieszczenia drzew w obrębie tych powierzchni mogą dostarczyć wartościowych informacji o strukturze badanego drzewostanu. Podano również przykłady analiz wykonanych dzięki wykorzystaniu powszechnie dostępnego oprogramowania.

Słowa kluczowe: funkcja $K(t)$ Ripleya, rozmieszczenie drzew, rozmieszczenie niejednorodne

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